The Minimal Polynomial :→

Let $A = [a_{ij}]_{n \times n}$

$m_A(x) = a_0 + a_1 x + \cdots a_{r-1} x^{r-1} + x^r \qquad r \leq n$

s.t. $m_A(A) = 0$

$\Rightarrow a_0 I + a_1 A + \cdots a_{r-1} A^{r-1} + A^r = \bar{0}$

& for any $p(x)$; $\deg p(x) < r$

$p(A) \neq \bar{0}$

then $m_A(x)$ is called the Minimal polynomial of A.

Observation :→

1. If $\deg m_A(x) = 1$

$\Leftrightarrow m_A[x] = x + a \quad , \quad a \in F$

$\Leftrightarrow A + a I_n = 0$

$\Leftrightarrow A = -a I_n$

$\Leftrightarrow A$ is scalar → [illegible]

2. if for any $F(x) = a_0 + a_1 x + \cdots a_k x^k \in R[x] ; F[x]$

s.t. $f(A) = \bar{0}$

$\Rightarrow m_A[x] \mid f(x)$

$\Rightarrow m_A(x) \mid C_A(x)$

$f(x) = m(x) \cdot q(x) + r(x)$

$f(A) = m(A) \cdot q(A) + r(A)$