

$$\text{Let } \alpha = -3 + \sqrt[3]{3}$$

$$(\alpha + 3) = \sqrt[3]{3} \quad \cdot \text{ cube both side}$$

$$(\alpha + 3)^3 = 3$$

$$\alpha^3 + 27 + 9\alpha(\alpha + 3) = 3$$

$$\alpha^3 + 9\alpha^2 + 27\alpha + 24 = 0$$

This polynomial is irreducible over \mathbb{Q} by the rational root theorem. So it is minimal polynomial.

$$m(\alpha) = \alpha^3 + 9\alpha^2 + 27\alpha + 24 \quad \underline{\text{Ans}}$$

$$\text{Degree} = 3 \text{ over } \mathbb{Q}$$
