# MATH1300C: Differential Calculus with Applications

**Solution to Term Test 2**

**Note:** The methods are not unique, and the final answers might be in different but equivalent forms.

**1.** Give the statement of Intermediate Value Theorem. (**5 pts**)

If $f(x)$ is continuous on $[a,b]$, and assume that $f(a) < f(b)$, then for any $y \in (f(a), f(b))$, there exists $c \in (a,b)$ such that $f(c) = y$.

**2.** Prove that if $f'(x) = g'(x)$, then $f(x) = g(x) + c$ for some constant $c$. (**10 pts**)

Let $h(x) = f(x) - g(x)$, then $h'(x) = f'(x) - g'(x) = 0$. Suppose $h(x)$ is not constant, then there exists $x_1 < x_2$ such that $h(x_1) \neq h(x_2)$. Then, by mean value theorem, there exists $c \in (x_1, x_2)$ such that $f'(c) = \dfrac{h(x_2) - h(x_1)}{x_2 - x_1} \neq 0$, contradicts to $h'(x) = 0$ for all $x$. Therefore, $h(x)$ is a constant function i.e. $f(x) = g(x) + c$ for some constant $c$.

**3.** Find the derivative of the following functions.

(a) $f(x) = (\cos 2x)^{1/x^2}$ (**5 pts**)

$$f'(x) = \left( -\frac{2\ln(\cos 2x)}{x^3} - \frac{2\sin 2x}{x^2 \cos 2x} \right) (\cos 2x)^{1/x^2}$$

(b) $f(x) = \sin^{-1}(2\sin x)$ (**5 pts**)

$$f'(x) = \frac{2\cos(x)}{\sqrt{1 - 4\sin^2 x}}$$

**4.** Find the following limits.

(a) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x - \tan x}$ (**5 pts**)