

Writing an Equation in Vertex Form by Completing the Square

We want to write an equation in vertex form $y = a(x - h)^2 + k$, by using a process called completing the square. We will start with the standard form of the quadratic equation, $y = ax^2 + bx + c$ and then rewrite the equation in vertex form.

EXAMPLE #1

Write $y = x^2 + 20x - 9$ in vertex form.

1. Add 9 to both sides, so the constant is eliminated from the right hand side.

$$y + 9 = x^2 + 20x - 9 + 9$$

$$y + 9 = x^2 + 20x$$

2. Now complete the square on the right hand side.

- Since $b = 20$, then $\left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2} \cdot 20\right)^2 = (10)^2 = 100$
- So completing the square, we must add 100 to both sides, because we are dealing with an equation.

$$y + 9 + 100 = x^2 + 20x + 100$$

$$y + 109 = x^2 + 20x + 100$$

3. We also know how to write the right side of the equation in factored form.

- Again, we know $b = 20$, so

$$y + 109 = \left(x + \frac{1}{2} \cdot 20\right)^2$$

$$y + 109 = (x + 10)^2$$

4. Now the equation is written in vertex form.

- $y = (x + 10)^2 - 109$, with a vertex of $(-10, -109)$