

Math Lab: Investigating the Quadratic Formula

Standard form of a quadratic equation is $ax^2 + bx + c = 0$. We just learned that any quadratic equation can be solved by completing the square. So we will complete the square using standard form to create a formula that will solve any quadratic equation!

Work	Steps in the process
$ax^2 + bx + c = 0$	Isolate the variable terms; move the constant to the right.
	Divide every term by the leading coefficient.
	Find the value to complete the square by taking half the coefficient of x and squaring it.
$\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$	Complete the square. Don't forget to add the same value to both sides of the equation!
$x^2 + \frac{b}{a}x + \frac{1}{4} = -\frac{c}{a} + \frac{1}{4}$	Factor the perfect square trinomial on the left.
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{1}{4}$	Find a common denominator on the right.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} + \frac{-c}{a}\left(\frac{1}{4}\right)$	Combine the two fractions with a common denominator into one fraction by adding the numerators.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Take the square root of both sides.
$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$	Use the quotient of radicals property to separate the right side into two radicals.
$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$	Solve for x by subtracting $\left(-\frac{b}{2a}\right)$ from both sides. Simplify the radical in the denominator on the right.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Combine the two fractions with a common denominator into one. This is the Quadratic Formula!
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	