Proof of Theorem:

$I \subseteq F[x]$:

$I = \{ q(x) \mid q($

① If $p, q \in$

then

$(p+q)($ ... $= 0$

② If

(rp)

Now $0, p_A \in I$.

so let $m_A \in I$ with smallest degree (monic, $\neq 0$).

If $q \in I$, $g = \gcd(m_A, q)$
$\neq 0$

Then BI: $\exists\, s(x), t(x)$ s.t.

$s(x) m_A(x) + t(x) q(x) = g(x)$.

$x \to$ ... $+ \; 0 = g(A)$.

$\Rightarrow g \in I$ and $0 < \deg g \leq \deg m_A$.

so $m_A = g$ and $m_A \mid q$. ✓