Problem 3. Consider the matrix

$$A = \begin{bmatrix} -9 & 16 & 8 \\ -7 & 12 & 5 \\ 2 & -3 & 0 \end{bmatrix}.$$

The minimal polynomial of this matrix is $\mu_A(x) = x^3 - 3x^2 + 3x - 1$, i.e.

$$\mu_A(A) = A^3 - 3A^2 + 3A - I = 0$$

(and there is no lower degree polynomial that works). In this problem you will compute A^{110}.

Using division with remainder, we may write

$$x^{110} = q(x)\mu_A(x) + r(x)$$

for some polynomials q, r, where $\deg r \leq 2$. If we can determine the polynomial r, then instead of computing A^{110}, we can just compute $r(A)$, because $\mu_A(A) = 0$. In our situation, $\mu_A(x) = (x-1)^3$, so the above equation is

$$x^{110} = q(x)(x-1)^3 + r(x).$$