
Good morning Class! Today we are going to learn how to find the polynomial equation of a graph using 5 examples.
Sounds like fun sir!
First class, let's look at the shape of the graph on our smartboard. This looks like a quartic function since its shaped like a"W". So, we know it will be a polynomial equation to the 4th degree.
Our next step is to look at the end behavior of the graph which is going up, so we know there is a positive lead coefficient. The graph cuts through the x axis twice, so we know the root has an odd multiplicity, in this case 1. It is also tangent to the x axis at one point, so that root has an even multiplicity, or 2.
Let's start to build our equation. The x-intercepts are (-1,0), (1,0), and (5,0); and the x-intercept of (1,0) is tangent to the x-axis, so we know it has a multiplicity of 2. So our equation is: $f(x)= a(x-1)^2 (x+1)(x-5)$
We know the y-intercept is (0, -10), so we can solve for "a" as follows: $-10=a(0-1)^2(0+1)(0-5)$ $-10=a(1)(1)(-5)$ $-10=-5a$ So, a=2
Finally, plug your 2 back into your original equation. So, our final equation is $f(x)=2(x-1)^2(x+1)(x-5)$
That's a little confusing, let's try some more!
We will practice a few more, that way you all can be masters at it!
Let's look at this one. The shape of this graph tells us it will be a quintic function, so the equation will have a degree of 5.
Now let's look at the end behavior. We know there is a negative lead coefficient because the graph ends downward. The graph cuts through the x-axis 3 times, so the roots will have an odd multiplicity of 1, 1 and 3. Since its a quintic function, the degree of the roots must add up to 5.
We know our x- intercepts are (2,0), (0,0), and (-2,0) and all are intersecting (vs tangent) the x-axis, so each root is an odd multiplicity, with the root of 2, and -2 occurring once and root 0 occurring 3 times. Our equation then will be: $f(x)=a(x+2)(x-0)^3(x-2)$
A point on our graph we know is (1,9), so we plug it in to our equation to solve for a: $f(9)= a (1+2)(1-0)^3(1-2)$ $9=a(3)(1)(-1)$ $9=-3a$ $a=-3$
We then get the following equation for the quintic graph: $f(x)=-3(x-0)^3(x+2)(x-2)$
I know it might still be a little confusing so don't worry! Let's go outside and get some fresh air, and practice a few more.
Awesome! I could use some fresh air.
Let's do a few more before lunch!
This is an easy one. Based on the graph, it is a parabola, so a quadratic equation with degree of 2.
As we see this graph has a positive end behavior. It also cuts through the graph twice. Our x intercepts will be (-3,0) and (3,0)
We now can start our equation. Both these intercepts cut through the graph so will have multiplicity of one. $f(x)=a(x+3)(x-3)$
Let's now plug in our y- intercept (0,-9) $-9=a(0+3)(0-3)$ $-9=-9$ $a=1$ So our leading coefficient is 1.
Finally plug in a=1 to our equation and we get: $f(x)=1(x+3)(x-3)$
Good job everyone! I can see everyone is getting it!
This one we can tell will be a cubic function based on its "s" shape curve and therefore will have a degree of 3.
This graph has a positive end behavior and only 1 x- intercept, which is at (1,0). Since it's a cubic function, we know the root of 1 will have to have a multiplicity of three. So the function will be: $f(x)=a(x-1)^3$
Now we can use the y-intercept at (0, -2) to solve for "a". $f(x)=a(x-1)^3$ $-2=a(0-1)^3$ $-2=-1a$ $a=2$
We now plug 2 for "a" back in our equation to get: $f(x)=2(x-1)^3$
We still have 10 mins before lunch, let's do one more, and call it a day.
Sounds like a plan, 1 more it is! Let's head back inside though.
This one is a little more challenging. It is a quartic function, so the shape and we know the leading coefficient will be negative, since its end behavior is negative. This function will have a degree of 4.
Let's look at the x-intercepts of -2 and 2. Since this is a quartic function, the degree is 4 and the multiplicity is odd since the points cut through the x-axis vs being tangent to the x-axis. So, looks like the root of -2 or (x+2) has a multiplicity of 3 and root 2 has a multiplicity of 1 (x-2).
Let's start the equation then using this information: $f(x)=a (x+2)^3(x-2)$
Our y-intercept is (0,32) so we plug it in to solve for "a". $32=a(0+2)^3(0-2)$ $32=a(8)(-2)$ $32=-16a$ $a=-2$
Finally we plug a=-2 back into our equation to get the following final equation from our quartic graph: $f(x)=-2(x+2)^3(x-2)$
That's it for today class, have a great rest of the day ! You did great!