ECE 3640

Lecture 3 | Building blocks for signals: Vector spaces

Objective: To learn about how to construct signals (and other things) from basic building blocks as a preparation for studying Fourier series.

An informal example

Before getting to that formality, an analogy will be presented.
We will consider making things out of a bunch of building blocks. The things might be, for example cakes, shampoo, or signals. For the case of cakes, the set of possible ingredients might look like this:

Ingred. symbol	Name
i_1	white our
i_2	wheat our
i_3	granulated sugar
i_4	Sweet and Low
i_5	Powdered sugar
i_6	Baking soda
i_7	Baking power
i_8	Hershey's cocoa
i_9	egg
i_{10}	milk
i_{11}	water
i_{12}	vegetable oil
$\vdots$	$\vdots$
i_N	Xanthum gum.

The ingredients are denoted by i_k. The mixture for the cake batter consists of certain amounts of each ingredient. The ingredients for a cake might be, for example: 500 ml white our, 300 ml granulated sugar, 2 eggs, 20 ml vegetable oil, 200 ml water, and 10 ml baking powder. (Hint: don't try to make this at home!). Assuming that the quantities in the table above are placed in the correct units (\normalized"), this recipe could be written as follows:

$$\mathbf{c} = 500\mathbf{i}_1 + 300\mathbf{i}_3 + 10\mathbf{i}_7 + 2\mathbf{i}_9 + 200\mathbf{i}_{11}20\mathbf{i}_{12}.$$

The set of all possible cakes forms a \vector space". If the set of ingredients is able to make every element in the space (i.e. every cake), the said of ingredients is complete. Notice that a complete cake space is not necessarily able to make everything else: we could not, for example, make every possible shampoo with the set of ingredients to make cakes. (We don't have, for example, any aloe in our list above, or even any FD&C Red # 21.)

Several interesting questions now arise. Given a cake, is it possible to determine the quantities of each ingredient that goes into it? (This is the analysis question.) Suppose that we only want to use a certain subset of the ingredients (say, we have run out of ingredients and don't want to run to the store). What is the best approximation to a desired cake that we can make? What is the error between the desired cake and the cake we actually get? (This is the approximation question.) Obviously, some of these questions don't make a lot of sense when applied to cakes. However, these kinds of things will be very applicable when it comes to analyzing signals, which may also be built up from a set of building blocks.