DISCRETE MATHEMATICS, WORKSHEET 1

Due on Wednesday, February 4th at 14:00. You may submit up to 24 hours late with a penalty of 5% on your mark.

The worksheet has 8 problems:

(1) Problems marked with (∗∗) are to be submitted and you **may not** ask for help from anyone except myself.
(2) Problems marked with (∗) are to be submitted and you **may** ask for help from anyone you want.
(3) Problems that have no asterisk symbol will not be marked. It is up to you whether you want to do them or submit them, however they will be considered as known problems for exams or midterm purposes.

Solutions to all problems will be available three days after the deadline on Study Direct.

Problem 1. Show that $\sqrt{3}$ is irrational.

Problem 2.
(1) Find $d = (a, b)$ in the following cases using the Euclidean Algorithm:
    (a) $a = 7469$, $b = 2464$;
    (b) $a = 1109$, $b = 4999$.
    In both cases, express d as a linear combination of a and b.
(2) Compute in any way you like
    (a) $[7469, 2464]$;
    (b) $[35, 45]$.

Problem 3 (∗).
(1) Suppose d, a, b in $\mathbb{N}$ satisfy
    (a) $d \mid a$, $d \mid b$;
    (b) $d = ax + by$ for some x, y in $\mathbb{Z}$.
    Show that $d = (a, b)$.
(2) Let $a, b \in \mathbb{N}$ and let $S = \{ax + by \mid x, y \in \mathbb{Z}\}$. If d is the least positive integer in the S, show that $d = (a, b)$. How many elements of S are common divisors of a and b ?

Problem 4 (∗).
(1) Show that, if $d \mid n$, then $a^d - 1 \mid a^n - 1$, for a, d, n in $\mathbb{N}$.
(2) Show that $(a^n - 1, a^m - 1) = a^{(n,m)} - 1$.
    Hint: use previous problem

Problem 5 (∗). Show that for a, b, $c \in \mathbb{N}$,
$$[(a, b), (a, c)] \mid (a, [b, c])$$

Problem 6 (∗∗). You have an unlimited supply of water, a drain, a large container and two jugs with capacity 7 and 9 litres respectively. How can you arrange to put exactly one litre of water in the container?