Sablas, Edizar L. September 22, 2022

BSEE-2 DAY

Seatwork: Chapter 2 Variable Separable

Instruction: Obtain the particular solution of the given differential equation.

1. Given: $(2a^2 - r^2)dr = r^3 \sin\theta d\theta$

when: $\theta = 0$, $r = a$

Solution:

$$2a^2 - r^2 dr = r^3 \sin\theta d\theta$$

$$\frac{2a^2 - r^2}{r^3} dr = \frac{\cancel{r^3} \sin\theta d\theta}{\cancel{r^3}}$$

$$\frac{2a^2 - r^2}{r^3} dr = \sin\theta d\theta$$

$$\int \frac{2a^2}{r^3} - \frac{r^2}{r^3} dr = \int \sin\theta d\theta$$

$$\int \frac{2a^2}{r^3} dr - \int \frac{1}{r} dr = \int \sin\theta d\theta$$

$$2a^2 \int \frac{1}{r^3} dr - \ln|r| = -\cos\theta + c$$

$$\cancel{2}a^2\left(-\frac{1}{\cancel{2}r^2}\right) - \ln|r| = -\cos\theta + c$$

$$-\frac{a^2}{r^2} - \ln|r| = -\cos\theta + c$$

$$(-\ln|r| = \frac{a^2}{r^2} - \cos\theta + c) - 1$$

$$\ln|r| = \frac{a^2}{r^2} + \cos\theta + c$$

$$e^{\ln|r|} = e^{-\frac{a^2}{r^2}} \cdot e^{\cos\theta} \cdot e^c$$

$$r = e^{-\frac{a^2}{r^2}} \cdot e^{\cos\theta} \cdot c$$

$$r = ce^{-\frac{a^2}{r^2}} \cdot e^{\cos\theta}$$ General solution

$$r = ce^{-\frac{a^2}{r^2}} \cdot e^{\cos\theta}$$

$\theta = 0$, $r = a$

$$a = ce^{-\frac{a^2}{a^2}} \cdot e^{\cos 0}$$

$$a = ce^{-1} \cdot e^1$$

$$a = c\left(\frac{1}{\cancel{e}}\right) \cdot \cancel{e}$$

$$a = c(1)$$

$$a = c$$

Therefore Particular Solution is

$$r = ae^{-\frac{a^2}{r^2}} + \cos\theta$$

Instruction: Obtain the General solution of the given differential equation.

2.) Given: $a\,d\beta + \beta\,da + a\beta(3da + d\beta) = 0$

Solution:

$$a\,d\beta + \beta\,da = a\beta(3da + d\beta) = 0$$

$$a\,d\beta + \beta\,da = -a\beta(3da + d\beta)$$

$$a\,d\beta + \beta\,da = -3a\beta\,da - a\beta\,d\beta$$

$$a\,d\beta + a\beta\,d\beta = -3a\beta\,da - \beta\,da$$

$$a(db + \beta\,d\beta) = -\beta(3a\,da + da)$$

$$\frac{d\beta + \cancel{\beta}\,d\beta}{-\cancel{\beta}} = \frac{3a\,da + da}{a}$$

$$-\frac{d\beta}{\beta} - \frac{\cancel{\beta}\,d\beta}{\cancel{\beta}} = \frac{3\cancel{a}\,da}{\cancel{a}} + \frac{da}{a}$$

$$-\int \frac{d\beta}{\beta} - \int d\beta = \int 3\,da + \int \frac{da}{a}$$

$$-\ln|\beta| - \beta = 3a + \ln a + c$$

$$(-1)(-\ln|\beta|) = (\beta + 3a + \ln a + c) - 1$$

$$\ln|\beta| = -\beta + 3a + \ln a + c$$

$$e^{\ln|\beta|} = e^{-\beta + 3a + \ln a + c}$$

$$e^{\ln|\beta|} = e^{-\beta} \cdot e^{3a} \cdot e^{\ln a} \cdot e^c$$

$$\beta = e^{-\beta} \cdot e^{3a} \cdot a \cdot c$$

$$\beta = ace^{-\beta + 3a}$$

Therefor the General Solution is

$$\beta = ace^{-\beta + 3a}$$