

INTEGRATION BY PARTS: measure the effectiveness of your answer by the number of digits shown in the question.

For each derivative:

$$\begin{aligned} &= \int u \cdot v' \, dx \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \sin x^2 \, dx \quad \stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \cos x^2 \, dx \end{aligned}$$

$$\begin{aligned} &= \int \frac{u^2 v'}{u+v} \, dx \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right)^2 \cdot v' \quad \stackrel{\text{v'}}{=} \frac{10x}{3x^2+7} \, dx \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right)^2 \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{3}(3x^2+7)^2 \cdot v' \end{aligned}$$

$$\begin{aligned} &= \int \frac{u}{\sqrt{u^2 + v^2}} \, dx \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{x \ln(u^2 + v^2)}{u^2} \, dx \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} x \sqrt{u^2 + v^2} \cdot v' \end{aligned}$$

$$\begin{aligned} &= \int u \sqrt{v^2 + u^2} \, du \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{2}u^2 \sqrt{v^2 + u^2} \, du \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{2}u^2 \cdot v' \end{aligned}$$

$$\begin{aligned} &= \int u \sqrt{v^2 + u^2} \, du \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{2u} \sqrt{v^2 + u^2} \, du \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{2}u^2 \cdot v' \end{aligned}$$

$$\begin{aligned} &= \int \frac{u v' \, du}{u^2 + v^2} \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{u^2 + v^2} \, du \\ &\stackrel{\text{u}}{=} \left(\frac{1}{3}x^3 + 2x^2 \right) \cdot v' \quad \stackrel{\text{v'}}{=} \frac{1}{u} \, du \end{aligned}$$