Calculus 1

Definition of Derivative

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(x)$$

Lotz of different notations

$f(x)$	$f'(x)$
y	y'
$f(x)$	$\frac{d}{dx}f(x)$
$f(x)$	$\frac{dy}{dx}$

$\lim_{x\to a} f(x)$ exists *iff* $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

Equation of Tangent Line

$y - f(a) = f'(a)(x-a)$

Limit Laws

if $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then...

$$\lim_{x\to a} f(x)+g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a} f(x)-g(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

$$\lim_{x\to a} c f(x) = c \lim_{x\to a} f(x)$$

$$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \leftarrow \neq 0$$

$$\lim_{x\to a} (f(x))^n = \left[\lim_{x\to a} f(x)\right]^n$$

$$\lim_{x\to a} C = C$$

$$\lim_{x\to a} x = a$$

$$\lim x^n = a^n$$

Graphing

f DNE = discont. 0 = root

f' $- \to +$ min

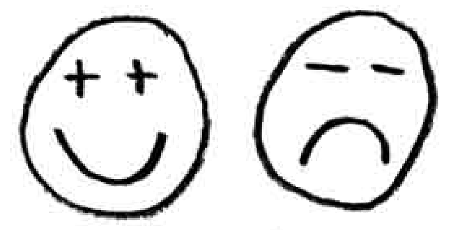
$+ \to -$ max

SLOPE

0 = critical point

(DNE = "critical number")

f''



Concavity

0 = point of inflection

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{x}{\sin x} = 1$$

ex $f(x) = 5x^4 + 3x + 2$

$5x^4$ = "end behavior model"

$$\lim_{x\to\infty} \frac{x^n}{x^{(>n)}} = 0$$

$$\lim_{x\to\infty} \frac{x^{(>n)}}{x^n} = \infty$$

$$\lim_{x\to\infty} \frac{Ax^n}{Bx^n} = \frac{A}{B}$$

Linearization

$L(x) = f(a) + f'(a)(x-a)$

$f(x) \approx L(x)$

ie use pretty # close to ugly # to get tangent line, plug in ugly #

Asymptotes

vertical: $\lim f(x) = \pm\infty$, $x \to a^+$ or $x \to a^-$

Horizontal: $\lim_{x\to\pm\infty} f(x) = L$ (finite)

$dy = f'(x)\,dx$

differentials

$L(x) = f(a) + f'(a)(\Delta x)$

as $\Delta x \to 0$, $\Delta A - dA \to 0$

$L(x) \approx f(a+\Delta x)$

$\approx f(a) + f'(a)\Delta x$

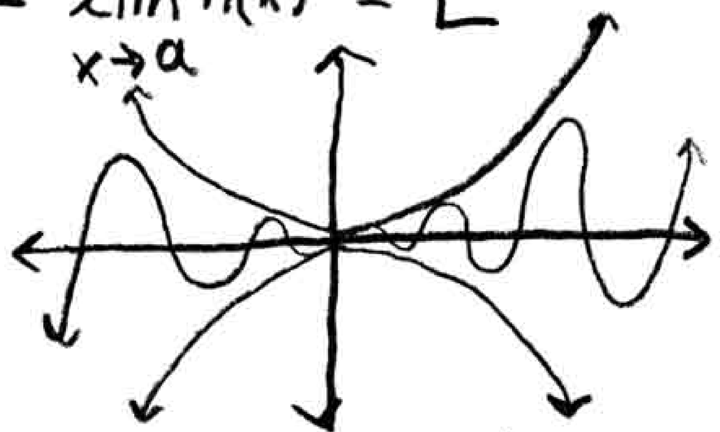
$\approx f(a) + dy$

if $f(x) \le g(x) \le h(x)$ when x is near a

and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$

then

$\lim_{x\to a} g(x) = L$



Sandwich

ex $-x^2 \le x^2 \sin(\frac{1}{x}) \le x^2$

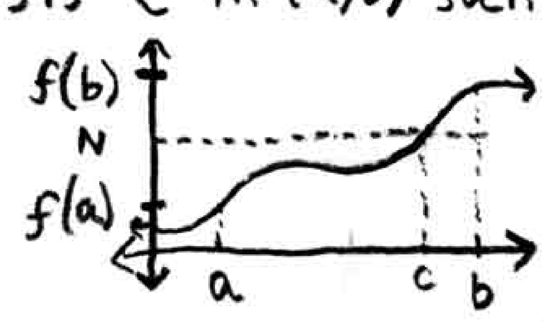
There is **Continuity** @ C if

1. $f(c)$ exists
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

Intermediate Value Theorem

if f is cont on $[a,b]$ and N is between $f(a)$ and $f(b)$, there exists c in (a,b) such that $f(c) = N$

$(f(a) \neq f(b))$



There is **Differentiability** @C if

1. f is continuous @ C
2. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists

Mean Value Theorem

if f is cont on $[a,b]$ and differentiable over (a,b) there exists at least one c where $f'(c) = \frac{f(b)-f(a)}{b-a}$

Newton's Method

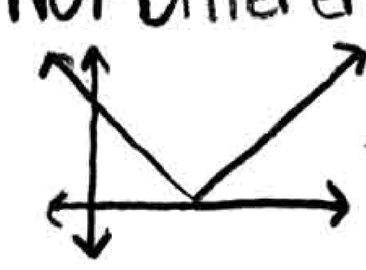
For Finding Roots (Approx)

1. Guess First Approx This is x_1 (or x_0)
2. Use First to find 2nd, 2nd to find 3rd, etc.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Not Differentiable if....

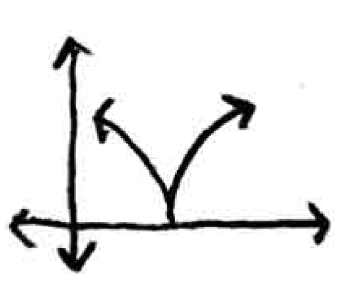
Corner



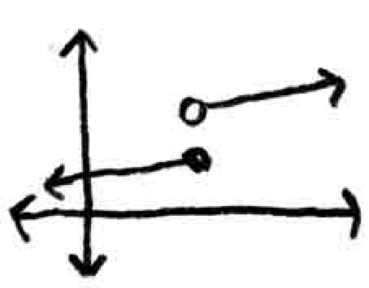
left′ ≠ right′ hand hand

slope of secant line approaches $+\infty$ or $-\infty$ from BOTH sides

Cusp



slope of secant line approaches $+\infty$ from one side $-\infty$ from other side



one or both sides $\frac{dy}{dx}$ = DNE (discontinuous)

L'Hopital's Rule

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

must have indeterminate form ie: 0/0 or ∞/∞

Derivative Rules

C	0
cu	$c\frac{du}{dx}$
$a+b$	$a'+b'$
$a-b$	$a'-b'$
ab	$ab' + ba'$
a/b	$\frac{ba'-ab'}{b^2}$
x^n	nx^{n-1}
e^u	$e^u\frac{du}{dx}$
a^u	$a^u \ln(a)\frac{du}{dx}$
$\ln(u)$	$\frac{1}{u}\frac{du}{dx}$, $u>0$
$\log_a(u)$	$\frac{1}{u}\frac{1}{\ln(a)}\frac{du}{dx}$
$\sin(u)$	$\cos(u)\frac{du}{dx}$
$\cos(u)$	$-\sin(u)\frac{du}{dx}$
$\tan(u)$	$\sec^2(u)\frac{du}{dx}$
$\csc(u)$	$-\csc(u)\cot(u)\frac{du}{dx}$
$\sec(u)$	$\sec(u)\tan(u)\frac{du}{dx}$
$\cot(u)$	$-\csc^2(u)\frac{du}{dx}$
$\sin^{-1}(u)$	$\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\lvert u\rvert<1$
$\cos^{-1}(u)$	$-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$,
$\tan^{-1}(u)$	$\frac{1}{1+u^2}\frac{du}{dx}$
$\csc^{-1}(u)$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$, $\lvert u\rvert>1$
$\sec^{-1}(u)$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$,
$\cot^{-1}(u)$	$-\frac{1}{1+u^2}\frac{du}{dx}$
$\sinh(u)$	$\cosh(u)\frac{du}{dx}$
$\cosh(u)$	$\sinh(u)\frac{du}{dx}$
$\tanh(u)$	$\text{sech}^2(u)\frac{du}{dx}$
$\text{csch}(u)$	$-\text{csch}(u)\coth(u)\frac{du}{dx}$
$\text{sech}(u)$	$-\text{sech}(u)\tan(u)\frac{du}{dx}$
$\coth(u)$	$-\text{csch}^2(u)\frac{du}{dx}$

parametric $\frac{dy}{dt} \Big/ \frac{dx}{dt}$

implicit example: y^3 → $3y^2\frac{dy}{dx}$