DEFINITION **Rates of Growth as $x \to \infty$**

Let $f(x)$ and $g(x)$ be positive for x sufficiently large.

1. f **grows faster than** g as $x \to \infty$ if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$$

or, equivalently, if

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0.$$

We also say that g **grows slower than** f as $x \to \infty$.

2. f and g **grow at the same rate** as $x \to \infty$ if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$$

where L is finite and positive.

⑥ $\displaystyle \lim_{x\to\infty} \frac{e^x}{x^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$

$\boxed{e^x \text{ grows faster than } x^2}$

⑦ $\displaystyle \lim_{x\to\infty} \frac{3^x}{2^x} = \lim_{x\to\infty} \left(\frac{3}{2}\right)^x = \infty$

$\boxed{3^x \text{ grows faster than } 2^x}$

⑧ $\displaystyle \lim_{x\to\infty} \frac{x^2}{\ln x}$

⑨ $\displaystyle \lim_{x\to\infty} \frac{\ln x}{x^{1/n}}$